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(with minor edits by K3PTO)

The concept of a decibel (dB) is understandably difficult and confusing for someone just being introduced to it. Combining specifications for gain, power, and voltage (and current, but not so often) that mix dB, dBm, dBW, watts, milliwatts, voltage, millivolts, etc., often requires converting back and forth between linear values and decibel values. This brief tutorial will help to clarify the difference between working with decibels and working with linear values.

Anxiety Alert: Using decibels involves working with logarithms.

Logarithms (logs) were first conceived of in the early 1600s by Scottish mathematician John Napier, as a tool for simplifying multiplication and division operations by converting them to faster and less error prone addition and subtraction operations, respectively. This is made possible because of the way multiplication of two numbers expressed as similar base numbers with exponents can be accomplished by merely adding the exponents together. Division of those same numbers is accomplished by subtracting the exponents. It is one of the laws of exponents, and looks like this:

$$x^a * x^b = x^{(a+b)} \quad \text{and} \quad x^a \div x^b = x^{(a-b)}$$

Using actual numbers as an example, where x = 10, a = 4, b = 1:

$$10^4 * 10^1 = 10^{(4+1)} = 10^5 \quad \text{and} \quad 10^4 \div 10^1 = 10^{(4-1)} = 10^3$$

$$10{,}000 * 10 = 100{,}000 \quad \text{and} \quad 10{,}000 \div 10 = 1{,}000$$

The law of exponents works for any base number, not just 10. To wit:

$$2^5 * 2^2 = 2^{(5+2)} = 2^7 \quad \text{and} \quad 2^5 \div 2^2 = 2^{(5-2)} = 2^3$$

$$32 * 4 = 128 \quad \text{and} \quad 32 \div 4 = 8$$

People tend to make fewer mistakes when adding and subtracting numbers, so the advantage of logarithms is apparent. Remember that logarithms were developed before automatic mechanical or electronic computers were available. A slide rule exploits the properties of logarithms for calculation, but that is a separate major topic.

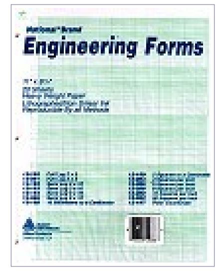
Those are simple examples, but hold for any base or exponent. In the absence of a calculator, in order to be useful for general application you need a table of numbers and their equivalent logarithms. Early log tables filled volumes, depending on the spacing between numbers (1.000, 1.001, 1.002, 1.003, vs. 1.0, 1.1, 1.2, 1.3, etc.). The good news for creators of logarithm tables is that only a single 'decade' of numbers (e.g., 1 through 10) is required since every preceding or succeeding decade is a simple multiple of a power of 10.

Note: I use base 10 in this discussion since that is the base of our common number system - hence the term 'common logarithm' for base 10 logs. You might have heard of natural logarithms, which uses the base of e, but e is not used very often when calculating scalar electrical power, voltage, and current quantities (although it is used when phase angles are included, i.e., Euler's identity). Natural logarithms are written as ln (x) without the 'e' subscript, whereas usually base 10 logarithms are written simply as log (x) without the 10 subscript; i.e., not $\log_e$ (x) or $\log_{10}$ (x), respectively.

Per a base=10 log table:

$$\log(100{,}000) = 5, \quad \log(10{,}000) = 4, \quad \log(1{,}000) = 3, \quad \log(10) = 1$$

The base-10 (common) logarithm of a number, then, is the exponent that 10 must be raised to in order to obtain that number. In other words, since 10 raised to the power of 2 is equal to 100 ($10^2 = 100$), the base-10 log of 100 is 2 ($\log_{10} 100 = 2$).



The exception and special case is $\log_x$ (0) = Undefined. That is so because there is no power to which you can raise any number and obtain 0 (zero). You can asymptotically approach zero, but you cannot get to zero. There will never be the number zero displayed on a log scale; they usually run from some power of 10 to some other power of ten. An example of log graph paper is shown on the right. It has 5 'cycles' or 'decades' of range. Note there is no zero on the y-axis.

This is the basic law of logarithms:

$$\log_c(a) = b, \quad \text{therefore} \quad c^b = a$$

Performing the same multiplications and divisions as done at the top of the page by using actual logarithms:

$$10{,}000 * 10 = 100{,}000 \quad \text{and} \quad 10{,}000 \div 10 = 1{,}000$$

$$4 + 1 = 5 \quad \text{and} \quad 4 - 1 = 3$$

That's fine, but what you end up with is the logarithm of the number you seek. Question: Except for a simple example like this, how do you get the answer you need? Answer: Look up the antilogarithm (antilog) of the result. In this case:

$$\text{antilog } 5 = 100{,}000 \quad \text{and} \quad \text{antilog } 3 = 1{,}000$$

A tougher, and more likely example with numbers that are not integer powers of 10, might look something like the following:

$$x = 1.28 * 3.70 * 0.559 * 26.4$$

$$\log(x) = \log(1.28) + \log(3.70) + \log(0.559) + \log(26.4)$$

$$\log(x) = 0.1072 + 0.5682 + (-0.2526) + 1.4216 = 1.8444$$

Since the logarithm of 'x' equals 1.8444, the antilog equals 'x,' which is 69.9

$$\text{Check: } x = 1.28 * 3.70 * 0.559 * 26.4 = 69.9$$

I used my calculator to look up the logs and antilogs for those numbers, but prior to 1972 when Hewlett Packard (HP) introduced their HP-35 scientific calculator, the average person without access to a corporate or university mainframe computer needed to use a log table to perform such calculations.

Who bothers to use logarithms today, you might ask? Lots of people, including me, quite often when calculating cascaded system parameters like noise figure (NF) and intercept points (IP). Simple addition and subtraction of gain dB and power dBm values don't work with NF and IP. The governing formulas use multiplication and division of linear gain and power values, which requires first converting dB and/or dBm to linear numbers (gain ratio and mW) using antilogs, performing the cascade calculations, and then converting the result back to dB and/or dBm using logs.

Not all system cascade operations require converting back and forth. For instance if only the total system gain and/or output power level is needed, then calculations can be carried out with either linear units (mW and multipliers) or logarithmic units (dBm and dB, respectively).

The Definition of 'dB' and 'dBm'

A decibel (dB) in electrical engineering is defined as 10 times the base-10 logarithm of a ratio between two power levels; e.g., P_{out}/P_{in} (gain, in other words):

$$dB = 10 * \log_{10} (P1/P2)$$

All gains greater than 1 are therefore expressed as positive decibels (>0), and gains of less than 1 are expressed as negative decibels (<0). Note that for cases most of us encounter, the linear ratio of P1/P2 must be a positive number (>0) since the logarithm of 0 is undefined and the logarithm of negatives numbers are complex (they contain both a real and an imaginary part). The dB value, though, can theoretically take on any value between $-\infty$ and $+\infty$, including 0, which is a gain of 1 [10 * log (1) = 0 dB].

'dBm' is a decibel-based unit of power that is referenced to 1 mW. Since 0 dB of gain is equal to a gain of 1, 1 mW of power is 0 dB greater than 1 mW, or 0 dBm. Similarly, a power unit of dBW is decibels relative to 1 W of power.

$$1 \text{ mW} = 0 \text{ dBm}$$

Accordingly, all dBm values greater than 0 are larger than 1 mW, and all dBm values less than 0 are smaller than 1 mW (see Fig. 1). For instance, +3.01 dBm is 3.01 dB greater than 1 mW; i.e., or 0 dBm + 3.01 dB = +3.01 dBm (2 mW). −3.01 dBm is 3.01 dB less than 1 mW; i.e., or 0 dBm + (−3.01) dB = −3.01 dBm (0.5 mW).

The following table gives some numerical examples so you can see the correlation between mW and dBm. The same set of values plotted on a logarithmic scale would produce a straight line. Because of the logarithmic relationship, the graph bunches the smaller values against the left vertical axis. A magnified version of the 0 to 1 mW region is inset for clarity.

	Power	
	mW	dBm
1 µW →	0.001	-30
	0.002	-27
	0.005	-23
10 µW →	0.01	-20
	0.02	-17
	0.05	-13
100 µW →	0.1	-10
	0.2	-7
	0.5	-3
1 mW →	1	0
	2	3
	5	7
10 mW →	10	10
	15	12
	20	13
	50	17
100 mW →	100	20
	200	23
	500	27
1 W →	1000	30

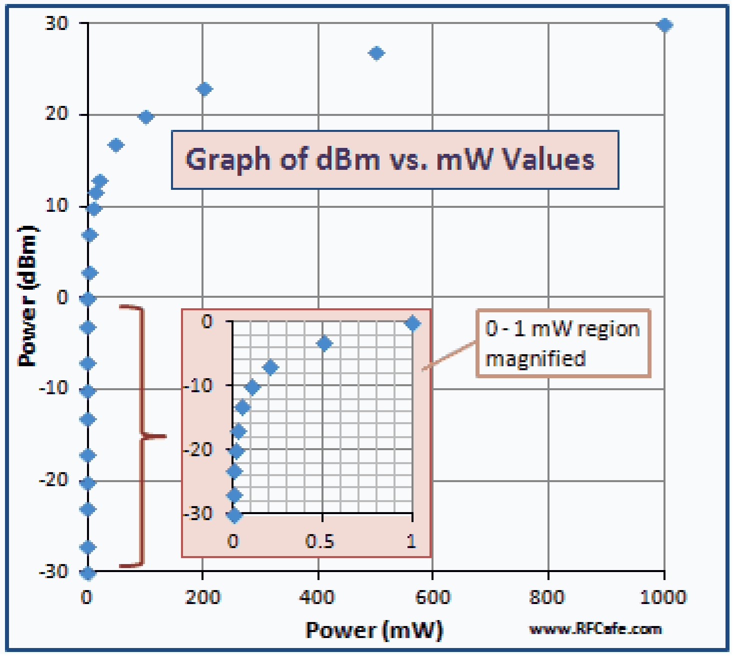


Fig. 1 - Graph of Power in Units of dBm vs. mW

Fig. 2 is a table and graph of dB vs. linear gain ratios similar to the dBm vs. mW in Fig. 1. Note that the numbers and curves are exactly the same; only the axis labels are changed. That is because dBm is a unit of power expressed in dB relative to 1 mW (0 dBm).

Output/Input Gain		
Linear		dB
1/1000	0.001	-30
1/500	0.002	-27
1/200	0.005	-23
1/100	0.01	-20
1/50	0.02	-17
1/20	0.05	-13
1/10	0.1	-10
1/5	0.2	-7
1/2	0.5	-3
1	1	0
2	2	3
5	5	7
10	10	10
15	15	12
20	20	13
50	50	17
100	100	20
200	200	23
500	500	27
1000	1000	30

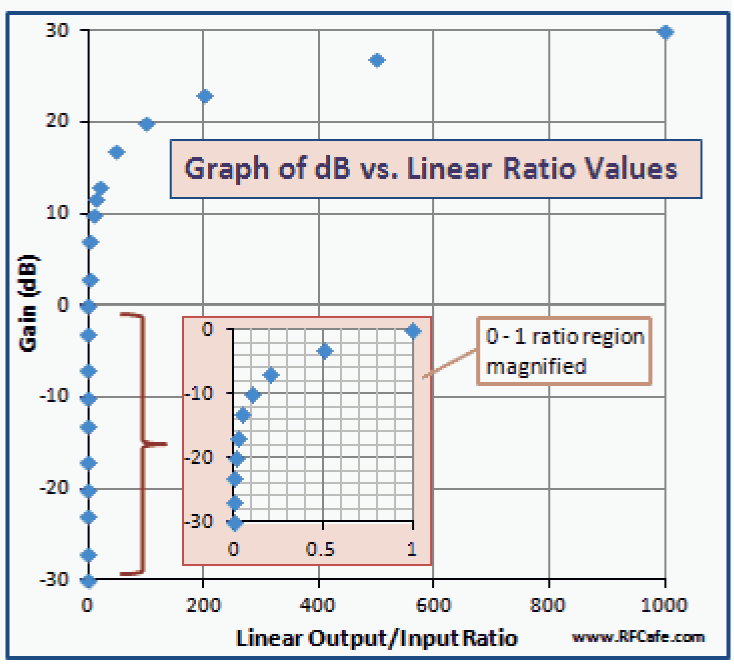


Fig. 2 - Graph of Gain in Units of dBm vs. Linear Ratio

Linear Gain (output/input ratio) vs. Logarithmic (decibels, dB) Gain

Fundamentally, gain is a multiplication (or division) factor. As an example, an amplifier might have a gain that increases the signal by a factor of 4 (i.e., 4x) from input to output (see Fig. 3). If a 1 mW (0 dBm) signal is fed into the amplifier, then 1 mW * 4 = 4 mW comes out. In terms of decibels, a factor of 4 is equivalent to 10 * log (4) = 6.02 dB, so 0 dBm input plus 6.02 dB of gain yields +6.02 dBm at the output.

$$1 \text{ mW} * 4 = 4 \text{ mW}$$

$$0 \text{ dBm} + 6.02 \text{ dB} = 6.02 \text{ dBm}$$

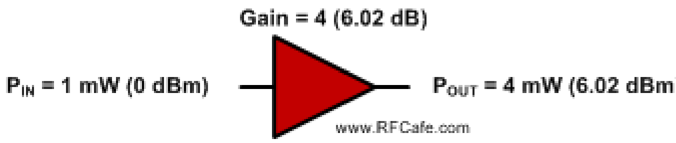


Fig. 3 - Single amplifier gain.

Combining Gains (linear and dB) w/Positive Values

If an amplifier with a gain of 4 is in series with a second amplifier with a gain of 6, then the total gain is 4 * 6 = 24. In terms of decibels, a factor of 6 is equivalent to 10 * log (6) = 7.78 dB, and a factor of 24 is equivalent to 10 * log (24) = 13.8 dB.

Just as 4 x 6 = 24 (linear gain), 6.02 dB + 7.78 dB = 13.8 dB (decibel gain).

If a 1 mW signal (0 dBm) is fed into the amplifier, then 4 mW comes out of the first amplifier, and 24 mW comes out of the second amplifier. See Fig. 4.

$$1 \text{ mW} * 4 * 6 = 24 \text{ mW}$$

$$0 \text{ dBm} + 6.02 \text{ dB} + 7.78 \text{ dB} = 13.8 \text{ dBm}$$

Fig. 4 - Cascaded dual amplifier gain.

Combining Gain and Loss (linear and dB)

This next example shows what happens when a gain < 1 (a loss) is encountered, where an attenuator with a gain of 1/6 is placed after the first amplifier instead of having a second amplifier. See Fig. 5.

4 * 1/6 = 2/3 (linear gain). Similarly 6.02 dB - 7.78 dB = −1.76 dB (decibel gain).

As with the previous example, if a 1 mW signal (0 dBm) is fed into the amplifier with a gain of 4, then 4 mW comes out. That 4 mW then goes into the attenuator with a linear gain of 1/6 and comes out at a power level of 4/6 mW (2/3 mW).

The total gain in this case is 4/6 = 2/3, so the output power will actually be less than the input power.

$$1 \text{ mW} * 4 * 1/6 = 2/3 \text{ mW} = 0.67 \text{ mW}$$

0 dBm + 6.02 dB + (-7.78 dB) = −1.76 dBm

Fig. 5 - Cascaded amplifier gain and attenuator.

Note that power levels greater than 0 dBm sometimes include the 'plus' sign (+) in order to emphasize that it is not negative. This is particularly so when power levels are displayed on a block diagram where both positive and negative values are present.

Summary

When making power measurements in the laboratory or in the field, most people find it easier to add and subtract gains and power levels than to multiply and divide gains and power levels. dB and dBm units make that possible. The important thing to remember is to never mix linear gain (ratio) units and wattage power (mW) units with logarithmic gain (dB) and power (dBm) units.

Quantities must be either in all linear or all decibel units. The following type of calculation is NOT allowed because it mixes linear values with logarithmic values.

12 mW + 34 mW + 8 mW + 20 dB

Supplemental Information on Logarithms

Logarithms of Products

A property of logarithms used implicitly above states the following, and is the basis for being able to add and subtract logarithm values instead of multiplying their linear equivalents.

$$\log(h*j) = \log(h) + \log(j), \quad \text{and} \quad \log(h/j) = \log(h) - \log(j)$$

therefore,

$$\log(h*j/k*m/n) = \log(h) + \log(j) - \log(k) + \log(m) - \log(n)$$

'h * j / k * m / n' might represent a cascade of components that have three devices (h, j, and m) each with gain >1 and two devices (k and n) each with a gain <1 (see Fig. 6). The total system gain can be calculated either by multiplying all the linear gain values together or adding all the decibel gain values together.

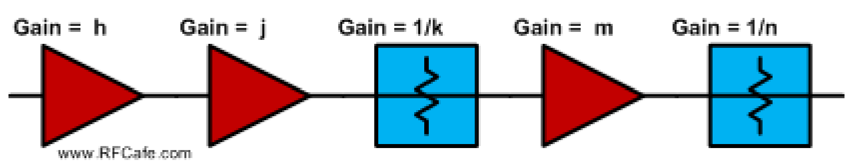


Fig. 6 - Cascaded components

See more on properties of logarithms and properties of exponents.

Logarithms of Exponents

The following is important for understanding why power gain in terms of power is 10 * log (P_{out}/P_{in}) dB, while power gain in terms of voltage is 20 * log (V_{out}/V_{in}) dB.

$$\log (c^f) = f * \log (c),$$

which is so because c^f is equal to c multiplied by itself 'f' times. For example, if f = 4:

$$c^f = c^4 = c * c * c * c$$

$$\log (c^4) = \log (c * c * c * c) = \log (c) + \log (c) + \log (c) + \log (c) = 4 * \log (c).$$

Power Gain Based on Power vs. Power Gain Based on Voltage

Power gain is P_{out}/P_{in}, and voltage gain is V_{out}/V_{in}. Power gain based on a power ratio in decibels is defined as 10 * log (P_{out}/P_{in}). Power gain in terms of voltage, is $[(V_{out}^2/R)/(V_{in}^2/R)]$, since per Ohm's law $P = V^2/R$. The 'R' in the denominators cancel leaving V_{out}^2/V_{in}^2 , which is equal to $(V_{out}/V_{in})^2$, as defined by the rule of exponents that says $a^c/b^c = (a/b)^c$. Hence:

$$10 * \log \left[\left(\frac{V_{out}}{V_{in}}\right)^2\right] = 10 * 2 * \log \left(\frac{V_{out}}{V_{in}}\right) = 20 * \log \left(\frac{V_{out}}{V_{in}}\right)$$

Important Note: Voltage gain in terms of voltage is 10 * log (V_{out}/V_{in}) dB, the same as with power gain in terms of power. It is only when power gain is expressed in terms of voltage that the 20 * log (V_{out}/V_{in}) dB equation applies. This is a common point of confusion.

Gain <1 (Loss) as Negative Decibels

No operation in mathematics is arbitrary, and that goes for why a signal power loss (gain <1) is portrayed as a negative value, and hence is subtracted during a cascade calculation. It is a simple demonstration, but worthy of mentioning.

$$\log (1/f) = \log (1) - \log (f) = 0 - \log (f) = -\log (f)$$

added by K3PTO

	dBm	mW	V @ 50ohms	mV @ 50ohms	uV @ 50ohms
1mW	0.0E+0	1.0E+0	223.6E-3	223.6E+0	223.6E+3
	-10.0E+0	100.0E-3	70.7E-3	70.7E+0	70.7E+3
	-20.0E+0	10.0E-3	22.4E-3	22.4E+0	22.4E+3
1uW	-30.0E+0	1.0E-3	7.1E-3	7.1E+0	7.1E+3
	-40.0E+0	100.0E-6	2.2E-3	2.2E+0	2.2E+3
	-50.0E+0	10.0E-6	707.1E-6	707.1E-3	707.1E+0
	-60.0E+0	1.0E-6	223.6E-6	223.6E-3	223.6E+0
	-70.0E+0	100.0E-9	70.7E-6	70.7E-3	70.7E+0
S9	-73.0E+0	50.1E-9	50.1E-6	50.1E-3	50.1E+0
S8	-79.0E+0	12.6E-9	25.1E-6	25.1E-3	25.1E+0
	-80.0E+0	10.0E-9	22.4E-6	22.4E-3	22.4E+0
S7	-85.0E+0	3.2E-9	12.6E-6	12.6E-3	12.6E+0
	-90.0E+0	1.0E-9	7.1E-6	7.1E-3	7.1E+0
S6	-91.0E+0	794.3E-12	6.3E-6	6.3E-3	6.3E+0
S5	-97.0E+0	199.5E-12	3.2E-6	3.2E-3	3.2E+0
	-100.0E+0	100.0E-12	2.2E-6	2.2E-3	2.2E+0
S4	-103.0E+0	50.1E-12	1.6E-6	1.6E-3	1.6E+0
S3	-109.0E+0	12.6E-12	793.4E-9	793.4E-6	793.4E-3
	-110.0E+0	10.0E-12	707.1E-9	707.1E-6	707.1E-3
S2	-115.0E+0	3.2E-12	397.6E-9	397.6E-6	397.6E-3
	-120.0E+0	1.0E-12	223.6E-9	223.6E-6	223.6E-3
S1	-121.0E+0	794.3E-15	199.3E-9	199.3E-6	199.3E-3

SWR	dB	Power Gain	Power Remaining	Cable	Cable Loss/100 ft @100MHz	Cable Loss/100 ft @450MHz
1.10	0.01	1.002	0.998	LDF5-50A	0.36	0.83
1.15	0.02	1.005	0.995	LDF6-50A	0.36	0.57
1.18	0.03	1.007	0.993	LMR-900	0.53	1.17
1.21	0.04	1.009	0.991	LDF4-50A	0.66	1.51
1.24	0.05	1.012	0.989	LMR-400	1.23	2.7
1.27	0.06	1.014	0.986	RG-8	1.9	2.8-4.5
1.29	0.07	1.016	0.984	RG-213	2.12	5
1.31	0.08	1.019	0.982	LMR-240	2.46	5.3
1.33	0.09	1.021	0.979	RG-6A	2.8	6.3
1.36	0.1	1.023	0.977	RG-8X	3.26	7.5
1.54	0.2	1.047	0.955	RG-58	4	9-13
1.70	0.3	1.072	0.933			
1.84	0.4	1.096	0.912			
1.98	0.5	1.122	0.891			
2.12	0.6	1.148	0.871			
2.26	0.7	1.175	0.851			
2.39	0.8	1.202	0.832			
2.53	0.9	1.230	0.813			
2.66	1	1.259	0.794			
4.10	2	1.585	0.631			
5.81	3	1.995	0.501			